Tuning and Temperament

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What is Tuning ?

Tuning is the process of setting the adjustable parts of a musical instrument so that when the instrument is played, it produces a pleasing sound, both on its own and with other musical instruments being played at the same time. There are two very distinct aspects of the tuning process.

Tuning the Overall Pitch

It is an obvious necessity that, when more than one instrument is to be played simultaneously, the players must adjust their instruments to the same basic pitch. This is the "tuning up" process that we all do very frequently.

"Tuning up" is usually done with reference to a single note. The internationally recognized pitch standard for modern performance was adopted at an International Standards Association conference in London in 1939 as **a = 440 Hz** This means that the note A above middle C on a piano is produced by a frequency of 440 vibrations per second. For performance of early music, the standard of **a = 415 Hz**, a semitone lower, is commonly used.

It is important to realize that this "tuning up" process only sets a single common pitch reference, each player has then to produce the other eleven notes of the musical scale relative to this reference pitch. The relative pitches of these notes are not fixed unless described within the context of a named temperament

Tuning the Intervals of the Musical Scale or Tempering

Depending on the type of instrument, the pitch intervals of the musical scale are set by the manufacturer of the instrument, the tuner of the instrument or by the performer, each having varying degrees of control. This process is known as tempering.

Instruments fall in to three main categories according to their capability to produce precise pitches.

Category 1 instruments are free to produce any note they wish within their compass. Examples are the human voice, unfretted string instruments and wind instruments with slides.

Category 2 instruments are constrained by their key mechanisms or individually tuned strings and do not permit alterations of pitch during performance. Examples of these include most keyboard instruments and harps. The pitch intervals of the musical scale of such instruments are set by the tuner of the instrument.

Category 3 instruments are those that use finger holes, valves, frets or tangents and allow limited pitch control in performance. Examples are woodwind and brass instruments, viols, guitars, bag-pipes, clavichords and hurdy-gurdies.

What Makes a Note ?

When a string, a solid object or a column of air vibrates it not only produces a fundamental note, it also produces a series of harmonic notes which are higher in pitch than the fundamental note and bear simple relationships to the fundamental note. Taking a vibrating string as an example, the first harmonic note we find, confusingly called the 2nd harmonic, is an octave higher and is equivalent to half of the string length, the next harmonic is a twelfth above the fundamental and equivalent to a third of the string length, the next is two octaves above the fundamental and equivalent to a quarter of the string length and so on ad infinitum. The first fifteen notes of the harmonic series starting at C are:

The number and relative intensity of the harmonic notes determines the characteristic sound of an instrument. Some wind instruments which are closed or partly closed at one end have little or no even numbered harmonics and consequently no octave harmonics, stopped diapason organ pipes and some notes on clarinets are good examples of this. On some bowed strings, the majority of the sound energy can be found in the harmonics with very little fundamental note energy. The human ear and brain can however still perceive the overall note as the fundamental pitch.

There is the further complication that short resonators give harmonics that are increasingly out of tune as you go up the harmonic series, hence the shrill character of some instruments like the violin and the hurdy-gurdy.

What Makes a Harmony ?

When two notes are played simultaneously, the resultant sound is sweet if the harmonic series from each note fit together with many of pitches of the two harmonic series coinciding.

What's The Problem ?

The harmonic series shows that the pitch ratio between a note and its octave is 2:1. The ratio between a note and its major third, e.g. C-E is 5:4, arguably the sweetest of all two note harmonies.

An examination of any musical keyboard would seem to indicate that three successive major thirds make up an octave. C-E then E-G# then G#-C. If this is true, then multiplying 5/4 x 5/4 x 5/4 should equal 2/1 which it clearly does not. The result is 125/64 which is significantly short of an octave.

Three successive pure major thirds don't make an octave, in fact the C we arrived at is flat by 42% of a semitone, almost half way between B and C. If we use the ratios from the harmonic series we clearly have a big problem in building a scale, we can't even get the octaves to come out correctly.

We can get the octaves right by de-tuning and sharpening each of the thirds by 14% of a semitone. By similar detuning we can get the ratios of all the other intervals to be identical starting from any note.

The end result of this exercise is called Equal Temperament and it gives the ability to play in any key with tolerable intonation.

The bad news is that we have mistuned all the musical intervals to some extent and in particular, all the major thirds are 14% of a semitone sharp and all minor thirds are 16% flat. These intervals are so far out of tune that the beat rates are high enough to be inaudible. Perfect fourths are 2% sharp and perfect fifths are 2% flat.

To quote Alden Hackman *"an advantage of equal temperament is that there are no "wolf" intervals, so one can play easily in every key. The disadvantage is that no key sounds particularly good. We've listened to pianos and guitars for so long that we're pretty much used to it - music just sounds like that……………. The hurdy-gurdy is a good place to experiment with temperament, since it's a whole lot easier to move the tangent than to move the frets around on a guitar."*

On the Hurdy-Gurdy we don't need to play in all 24 possible keys so why sacrifice the harmonic qualities by using an inappropriate temperament. We can easily set a pure temperament by adjusting the tangents and our instrument will play very sweetly in most of the commonly used major and minor keys.

Ratios are not the easiest things to grasp so we need a more convenient way of expressing musical intervals.

A Better Way to Describe Intervals

When we need to combine musical intervals (frequency ratios) we must multiply them together. If we use logarithms we can add and subtract them which is much easier.

The logarithmic unit of measurement is the cent and it is independent of pitch. There are 1200 cents in an octave and 100 cents in each equally tempered semitone.

The interval in cents Ic between 2 notes of frequencies f_1 and f_2 is given by
$Ic = 1200 \log_2(f_1 / f_2)$ or $Ic = 3986.314 \log_{10}(f_1 / f_2)$

The frequency ratio of an interval of Ic cents is $f_1 / f_2 = \text{alog}_{10}(Ic/3986.314)$
If we go back to our pure major third which has a 5/4 ratio we can calculate the cent value of this interval as $3986.314 \log_{10}(5/4)$ which equals 386.314. This can also be thought of as 1 Pure Major Third = 3.86314 Equally Tempered Semitones which again is 14% of a semitone narrower than an equally tempered major third.

Most chromatic electronic tuning devices are calibrated in cents and usually show the equally tempered pitch as a center zero with minus or plus 50 cents either side to show flatness or sharpness. You can use these devices to set other temperaments if you know the differences in cents between each semitone of the new temperament and equal temperament.

The Differences Between Equally Tempered and Pure Notes

The following table gives the values in cents for Equal and Just (pure interval) temperament starting with an open G string.

Keyboard Note	Equal Temperament in Cents	Just Temperament in Cents		Adjustment in Cents Relative To Equal Temperament	Frequency Ratio To G
G	000	000.00	G	0	1 : 1
Ab	100	111.73	Ab	+12	16 : 15
A	200	203.91	A	+4	9 : 8
Bb	300	315.64	Bb	+16	6 : 5
B	400	386.31	B	-14	5 : 4
C	500	498.04	C	-2	4 : 3
Db	600	590.22	Db	-10	45 : 32
D	700	701.96	D	+2	3 : 2
Eb	800	813.69	Eb	+14	8 : 5
E	900	884.36	E	-16	5 : 3
F	1000	996.09	F	-4	16 : 9
F#	1100	1088.27	F#	-12	15 : 8
G	1200	1200.00	G	0	2 : 1

Practical Considerations for the Hurdy-Gurdy

Most Hurdy-Gurdies will have been manufactured with the key spacing set for equal temperament. To achieve the minimum tangent deflection, particularly from open up to B where there are significant tangent adjustments away from the nut, it can be useful to move the nut slightly back to reduce the deflections.

Another point to note is that as the intervals are pure, slight maladjustment of the tangents that produce notes that match the drone strings or slight mistuning of the drones can be more noticeable. When tuning in G for example it is advisable to check the d key against the trompette on its own and similarly check the c key when using a C trompette.

The tuning data given for a G / C instrument can be used for a D / G instrument without modification, simply treat each key as it looks rather than what it plays, in other words set the tangents in exactly the same way as if it were a G / C instrument.

Other Temperaments

There are many other temperaments used on keyboard instruments. They all work in different ways but the end result is to distribute the intonation errors in such a way that the damage is done to infrequently used keys while keeping the commonly used keys as pure as possible. There is a very useful side effect of introducing quite strong "key color" contrasts. When one contrasts the sound of early keyboard music played on an equally tempered instrument with the same music played on an instrument set to a contemporary temperament, the difference is very marked and one realizes that the composer had deliberately used the "key colors" as an integral part of the composition.

Other non-equal temperaments generally allow one to play in wider range of keys than required on the hurdy-gurdy and the deviations from equal temperament can be even larger resulting in extreme tangent offsets. Just temperament is intolerable on a conventional keyboard instrument but very suitable for a hurdy-gurdy.

More Technical Information

The only harmonic error discussed above is the mismatch between 3 consecutive major third and an octave. There are other well known and important errors with delightfully quaint names which I will list below for the sake of completeness.
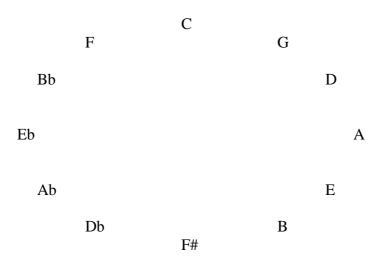
3 major thirds don't make an octave

1 octave	= 1 x 1200	= 1200
3 major thirds	= 3 x 386	= 1158
	Difference	= 42 cents

This difference is known as a ***Diesis***

12 perfect fifths don't make 7 octaves

12 perfect fifths	= 12 x 702	= 8424
7 octaves	= 7 x 1200	= 8400
	Difference	= 24 cents

This difference is known as the ***Ditonic or Pythagorean Comma***

The ***Ditonic Comma*** is a very important discrepancy used in deriving equal temperament. When rising by 12 consecutive perfect fifths we make one visit to each of the twelve semitones of the scale before arriving at the note we started at. If we flatten each of the fifths by 2 cents (24/12) the pitch of finishing note will be an octave multiple of the starting note pitch. This will set an equally tempered octave. This sequence of notes is known as the ***circle of fifths***

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                         C
          F                          G

     Bb                                   D


   Eb                                        A


     Ab                                  E

        Db                      B
                 F#
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4 perfect fifths don't make 2 octaves + 1 major third

4 perfect fifths	= 4 x 702	= 2808
2 octaves + 1 major third	= (2 x 1200) + (1 x 386)	= 2786
	Difference	= 22 cents

This difference is known as the ***Syntonic or Didymus' Comma***

8 perfect fifths + 1 major third don't make 5 octaves

8 perfect fifths + 1 major third	= (8 x 702) + (1 x 386)	= 6002
5 octaves	= 5 x 1200	= 6000
	Difference	= 2 cents

This difference is known as a ***Schisma***

Bibliography

Pagham Charles A. 1986 *The Well Tempered Organ*, Positif Press
ISBN 0 906894 13 1

Meffen, Charles *A Guide to Tuning Musical Instruments* David & Charles

Hackman, Alden *email to the author, January 25, 2000*